Natural Sciences 102 -- Spring 2004
Exam #2, May 20, 2004

Name (PLEASE print legibly): Answer Key

General instructions:
- For essay and descriptive questions, please be complete, but concise. Answers should be limited to the space provided under the question.
- Please manage your time. First answer the questions you know best.
- **Each problem is worth 10 points.**
- Some formulae you may find useful:

1 (10)	2 (20)	3 (10)	4 (10)	5 (10)	6 (10)	7 (10)	8(10)	Total (80)

apparent magnitudes of objects 1 & 2 (I = intensity): $m_1 - m_2 = -2.5\log\left(\frac{I_1}{I_2}\right)$

apparent magnitude of the sun: -26.8

$1\ \text{pc} = 200{,}000\ \text{AU}$ speed of light: 300,000 km s^{-1}

Hubble's law: $\text{v} = H_0 d$ speed of sound: 700 miles per hour

speed of smell: 200 inches per minute

shift in wavelength due to motion (c = velocity of the wave): $\frac{\lambda}{\lambda_0} = 1 \pm \frac{\text{v}}{c}$

Inverse square law: intensity $= \left(\frac{\text{luminosity}}{4\pi \text{R}^2}\right)$

Luminosity of the sun: 10^{26} Watts

Luminosity of President Bush: 10^{-4} Watts

$\left(\frac{\text{distance}}{\text{pc}}\right) = \left(\frac{\text{seconds}}{\text{parallax}}\right)$ $1\ \text{Mpc} = 3\times 10^{19}\ \text{km}$

I. The Cosmological Distance Scale:

Three things you can measure about stars are that 1) they appear to have different brightness, 2) they appear to have different colors, and 3) they may changes in brightness. Give one example of how each of these properties is used in the construction of the distance ladder. Please describe each example.

(1) Stars appear to have different brightness:

An object's brightness (or intensity *I*) depends on both its intrinsic luminosity *L* and its distance *D* from us, through the inverse square law:

$$I = \frac{L}{4\pi D^2}.$$

Knowing the brightness of an object whose distance is known from another method (such as parallax) allows one to compute the intrinsic luminosity.

(2) Stars have different colors:

Knowing a star's color (its spectral type) in addition to its brightness allows one to construct the Hertzsprung-Russell (HR) diagram, showing color vs. brightness (as shown in Problem III). For stars in a single cluster (i.e., at approximately the same, known distance), the stars fall in a particular and recognizable pattern. Plotting the stars from a different cluster (thus at a different distance), the pattern will be displaced in brightness from the first. The difference in intensity allows one to use the inverse square law to determine the distance).

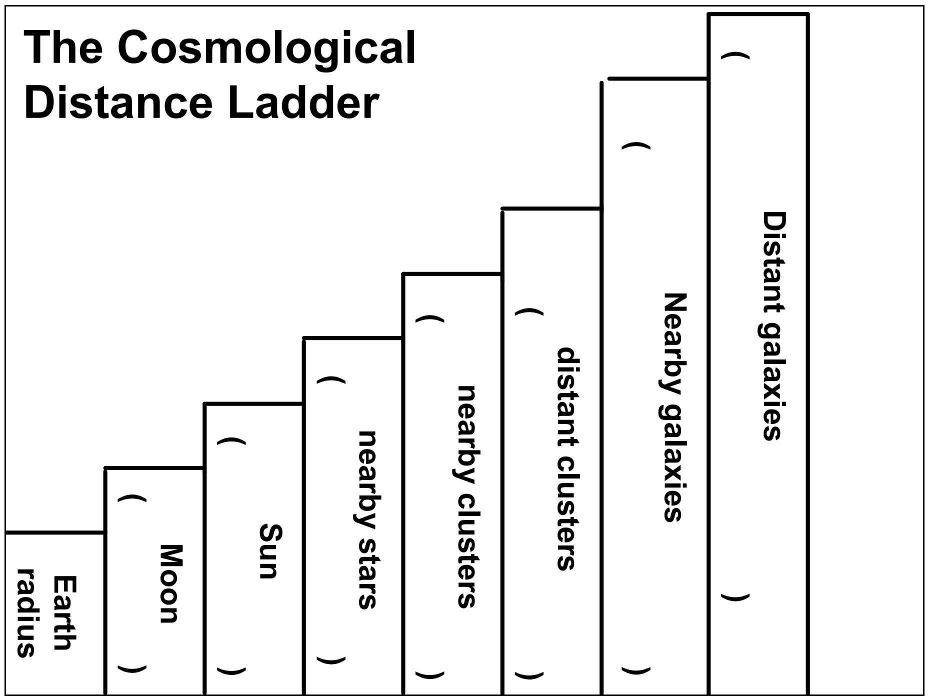
(Also technically acceptable was Hubble's Law. Due to the expansion of the universe, the spectrum (or "color") of a galaxy gets redshifted, making it appear that it is moving with some velocity v. The distance to the galaxy is then given from Hubble's Law: $D = H_0^{-1}\mathrm{v}.$)

(3) Stars may have changes in brightness:

RR Lyrae and Cepheid stars are easily identified by their pattern of variability (change in brightness). RR Lyrae stars all have approximately the same intrinsic luminosity, while Cepheids have a luminosity which is related to the measured period of variability. Having identified one of these stars in a cluster whose distance is known (by some other method), the exact value of the luminosity can then be used to use these as standard candles.

II. The Cosmological Distance Scale:

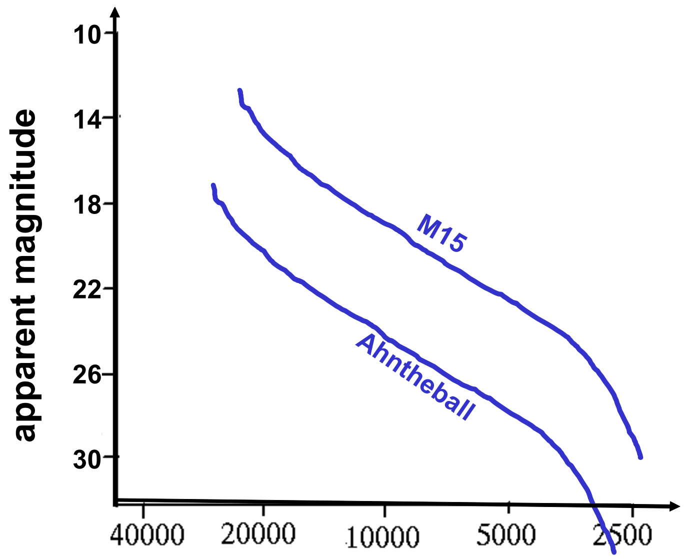
a) For each step in the distance ladder, fill in (in the parentheses) the method used to determine the distance. Choose from Cepheid, Faith-Based Determination, Geometry, H-R Diagram, Hubris, Hubble's Law, Parallax, RR-Lyrae, or Radio Dating.



The Cosmological Distance Ladder: Moon-Geometry, Sun-Geometry, nearby stars-Parallax, nearby clusters- H-R Diagram, distant clusters- RR-Lyrae, nearby galaxies-Cepheids, distant galaxies-Hubble's Law.

III. The H-R Diagram as a distance indicator:

a) Describe in words (use equations if you wish) how the H-R diagram can be used as a distance indicator.
b) The distance to the cluster M15 is known to be 1000 pc. Another cluster is discovered by Sein Ahn, named the Ahntheball cluster. Use the information from the H-R diagrams below to calculate the distance to Ahntheball.



3a) Many stars must be measured to establish a good HR diagram. Measuring only one star will not do. We assume that properties of globular clusters are similar and thus the overall trend of the HR diagram should be the same for all globular clusters. We compare HR diagram of Ahntheball with an HR diagram of a cluster whose distance is known, say M15. Plot intensity (apparent magnitude) in the y-axis. Ahntheball and M15 have the same **intrinsic** *L* luminosity, overall and at each particular temperature, but due to their distance *d*, their apparent luminosity does not appear to be the same.

$I_1 = \dfrac{L}{4\pi d_1^2}$, $I_2 = \dfrac{L}{4\pi d_2^2}$ since L_1=L_2, we can calculate the distance to Rocky-II:

$I_1 d_1^2 = I_2 d_2^2 \rightarrow d_1 = \sqrt{\dfrac{I_2}{I_1}}\, d_2$. The intensities can be determined by the

equation $m_1 - m_2 = -2.5\log\left(I_1/I_2\right)$. These two equations can be combined to give the complete equation relating distance to apparent magnitude at a given temperature, $m_1 - m_2 = -5\log\left(d_2/d_1\right)$.

3b) The magnitudes can be chosen at any temperature. The following is an example solution. At about 20,000 Kelvin, M15 has an apparent magnitude of about 15. Ahntheball has one at about 20. The distance to M15 is known to be 1000 pc. Therefore the distance to Ahntheball is given by $20 - 15 = -5\log\left(1000\text{ pc}/d_{\text{Ahntheball}}\right)$ where the 1 represents Ahntheball, and the 2 represents M15. So $d_{\text{Ahntheball}}$ is about 10,000 pc.

IV. Parallax:

a) What is annual stellar parallax? What moves, and what appears to move?
b) Draw a diagram indicating the parallax angle, the distance known by other means and the distance determined by parallax.
c) The annual stellar parallax of the star Chisholm 321 is measured to be 0.02 seconds of arc. What is its distance?

a) Annual Stellar Parallax is the apparent movement of a nearby star with respect to the more distant background stars. Maximum apparent displacement comes at roughly 6 month intervals, when the Earth is on opposite sides of the sun. In a sense, this parallax is an optical illusion--the stars appear to move, but it is the Earth which is really moving.

b) This should be some sort of diagram indicating that parallax is best measured from the Earth six months apart, in order to give the longest baseline (1AU) for the measurement. This diagram should look very similar to the diagram on page 7 of Dr. Kolb's lecture 8 (April 27).

c) You may certainly use the equation $\tan\alpha = 1\ \mathrm{AU}/D$, but this is more difficult than it needs to be. If you do use this equation, remember there is no need to convert your angle from degrees to radians. Provided your calculator knows whether it is getting degrees or radians, either may be entered in your calculator.

For small angles (and given that the closest star to the sun is at about 1 pc and, therefore, has a parallax angle of 1 arc second, all stellar parallax angles are small), you may use the simpler formula $\alpha = 1\ \mathrm{AU}/D$, which implies $D = 1\ \mathrm{AU}/\alpha$. Remember that, in this formula, distances will be in parsecs if the angle is in arc seconds. This problem just becomes:

$$D = 0.02 = 50\ \mathrm{pc}.$$

V. <u>Doppler Shift:</u>

a) Define the Doppler shift and explain how it can be used to determine velocity.
b) Justin Johnsen is driving his new pickup truck from Texas to Hyde Park. While passing through a trailer park outside of the small town of Rectalrash, Arkansas, he is pulled over for running a red light. He claims the light looked green to him because of the Doppler effect. The wavelength of red light is 6000 Angstroms and the wavelength of green light is 4500 Angstroms.
   1. In order for a red light to appear green, would Justin have to be traveling toward the light or away from the light?
   2. Estimate Justin's speed for a red light to appear green?

a) The Dopper shift is the shifting of emitted wavelengths (frequency) due to either the source or the observer moving. This holds for all forms of wave, including light and sound.

The change of wavelength is proportional to the velocity of the source or the observer; the faster the velocity, the greater the shift in wavelength.

b) 1. Justin is moving towards the light.

2. Since Justin is moving towards the object, the equation we need to use is

$$\frac{\lambda - \lambda_0}{\lambda_0} = -\frac{v}{c}$$

- Emitted wavelength (red): $\lambda_0 = 6000\ \text{Angstroms}$.
- Observed wavelength (green): $\lambda_0 = 4500\ \text{Angstroms}$.

The speed Justin was traveling is

$$\frac{4500 - 6000}{6000} = -\frac{v}{c}$$

$$\frac{-1500}{6000} = -\frac{v}{c}$$

$$0.25 = -\frac{v}{300{,}000\ \text{km s}^{-1}}$$

or

$$v = 0.25\ \times 300{,}000\ \text{km s}^{-1}$$

$$v = 75{,}000\ \text{km s}^{-1}.$$

VI. Cosmological Principle:

a) Please state the cosmological principle.
b) What are the implications of the cosmological principles for the center and edge of the universe?
c) Reconcile the observational fact that most galaxies are receding from us with the cosmological principle. Why wouldn't that fact make us special?

a)
The cosmological principle can be stated in two ways:

i. There is no preferred point in the Universe and there is no preferred set of points in the Universe
ii. The Universe in large scales is homogeneous and isotropic

b)
The cosmological principle implies that there is neither a center in the universe because that would be neither a special point nor a boundary because that would be a special set of points.

c)
The fact that we see almost all galaxies receding from ours, gives us the impression that we are the center of the universe. However, if that were true it would violate the cosmological principle. This observation is reconciled with the cosmological principle because no matter where in the universe one is, one would see all galaxies moving away from him. Thus, the universe is a democratic place, since every observer in it sees the same picture.

VII. Magnitudes:

a) Why is the magnitude scale based on a lograrithmic scale?
b) Which appears brighter, a star of 12th apparent magnitude or a star of 7th apparent magnitude?
c) The star Cheney-V has an apparent magnitude of m = –1. Another star, Kerry-7, is the same distance from us as Cheney-V, but has a luminosity of one hundred times the luminosity of Cheney-V. What is the apparent magnitude of Kerry-7?
d) Using the luminosity of President Bush given on the instruction page, calculate his magnitude if he was as far away as the sun.

a) It is based on the fact that our senses respond logarithmically to stimuli.

b) The 7th magnitude star is brighter, for the smaller the magnitude the brighter the source.

c) The formula for magnitudes is

$$m_K - m_C = -2.5\log\left(\frac{I_K}{I_C}\right) = -2.5\log\left(\frac{L_K/4\pi R_K^2}{L_C/4\pi R_C^2}\right).$$

Now we know $m_C = -1$, $I_K = 100 I_C$, and $R_K = R_C$

$$m_K - (-1) = -2.5\log\left(\frac{L_K}{L_C}\right)$$

$$m_K + 1 = -2.5\log(100) = -2.5\times 2 = -5$$

$$m_K = -6$$

d)

$$m_B - m_\odot = -2.5\log\left(\frac{I_B}{I_\odot}\right)$$

$$m_B - (-26.8) = -2.5\log\left(\frac{L_B/4\pi R_B^2}{L_\odot/4\pi R_\odot^2}\right)$$

Since $R_B = R_\odot$, $L_\odot = 10^{26}$ watts, and $L_B = 10^{-4}$ watts

$$m_B + 26.8 = -2.5\log\left(\frac{10^{-4}}{10^{26}}\right) = -2.5\log\left(10^{-30}\right)$$

$$m_B = -26.8 + 2.5\times 30 = -26.8 + 75$$

$$m_B = 48.2 \text{ (rather dim).}$$

VIII. Big Bang:

Please respond to the following "fact or fiction," and explain how you know the answer.

FICTION: If we look at the spectra of galaxies (as Hubble did) we see that the emission and absorption lines coming from them are redshifted, and that the farther away they are from us the more they are redshifted. We interpret this to be consistent with the expansion of space in the universe. This is the experimental base for Hubble's law of expansion of the universe $v = H_0 d$, where v is the velocity of the object, d its distance from us, and H_0 the Hubble constant. This law is also a consequence of the cosmological principle and the Big Bang theory. (In addition to this, recent measurements of supernova type Ia suggest that actually the universe is expanding at even a faster rate, due to the effect of a mysterious "dark energy" which fills all the universe.)